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which the whole theory is easily deduced and put in to shape for practical use.*

Those familiar with the history of this subject before the time of Clapeyron's discovery will at once recognize the simplicity of the above equations compared with the complicated method of Navier.

NOTE ON THE DIVISION OF SPACE.

BY PROF. HALL.

This question occurred to me several years ago in reading an account of Bertrand's method of treating the doctrine of parallel lines. I have not seen Prof. Cayley's solution, but of course so obvious a question could not be new. Lately I have found that this question and several kindred ones are very completely discussed by Steiner in the first Vol. of Crelle's Journal, published in 1826. For the number of parts into which n planes can divide space Steiner finds an expression which is eqnivalent to $\frac{1}{6}(n^3+5n+6)$; and he shows also that of these parts

$$\frac{(n-1)(n-2)(n-3)}{1\,.\,2\,.\,3} \text{ are limited,}$$

and $2+n(n-1)$ are unlimited.

CORRECTION OF AN ERROR IN BARLOW'S THEORY OF NUMBERS.

BY ARTEMAS MARTIN, ERIE, PA.

In *Barlow's Theory of Numbers*, page 299, it is stated that "the equation

$$x^2 - 5658y^2 = 1$$

has its least values as follows; viz.

$x = 166100725257977318398207998462201324702014613503$,

$y = \quad 698253616416770487157775940222021002391003072$."

I will show that these values are not correct, and then compute the true numbers.

The units figure of the square of Barlow's value of x is 9; the units figure of the square of his value of y is 4; the units figure of $5658y^2$ is 2, and $9 - 2 = 7$. If his values were correct the units figure of x^2 would be 1 greater

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*For a full presentation of ready methods for finding moments and reactions the reader may consult the *Journal of the Franklin Institute*, March and April, 1875.

than the units figure of $5658y^2$ unless the units figure of x^2 was 0, in which case the units figure of $5658y^2$ would be 9.

Let $A = 5658$, then $\sqrt{(5658)} = \sqrt{A} = r + \cfrac{1}{u_1 + \cfrac{1}{u_2 + \cfrac{1}{u_3 + \text{\&c.},}}}$

where r is the greatest integer contained in $\sqrt{A}$.

The last quotient of every complete period is $2r$. Let m be the number of quotients in a complete period, and $p_m \div q_m$ the last convergent in the first period; then, when m is even, $x = p_m$, $y = q_m$, and when m is odd $x = p_{2m}$, $y = q_{2m}$.

Let $\frac{\sqrt{(A)} + a_n}{b_n} = u_n + \text{\&c.}$ and $\frac{\sqrt{(A)} + a_{n+1}}{b_{n+1}} = u_{n+1} + \text{\&c.}$ be any two consecutive complete quotients, then

$$a_{n+1} = u_n b_n - a_n, \quad b_{n+1} = \frac{A - a_{n+1}^2}{b_n}.$$

If $p_n \div q_n$, $p_{n+1} \div q_{n+1}$ be any two consecutive convergents and u_{n+1} the quotient corresponding to $p_{n+1} \div q_{n+1}$, then

$$\frac{p_{n+2}}{q_{n+2}} = \frac{u_{n+1} p_{n+1} + p_n}{u_{n+1} q_{n+1} + q_n}.$$

$$\frac{\sqrt{(5658)} + 0}{1} = 75 + = r, \qquad \frac{r + 75}{33} = 4 + = u_1,$$

$$\frac{r + 57}{73} = 1 + = u_2, \qquad \frac{r + 16}{74} = 1 + = u_3,$$

$$\frac{r + 58}{31} = 4 + = u_4, \qquad \frac{r + 66}{42} = 3 + = u_5,$$

$$\frac{r + 60}{49} = 2 + = u_6, \qquad \frac{r + 38}{86} = 1 + = u_7,$$

$$\frac{r + 48}{39} = 3 + = u_8, \qquad \frac{r + 69}{23} = 6 + = u_9,$$

$$\frac{r + 69}{39} = 3 + = u_{10}, \qquad \frac{r + 48}{86} = 1 + = u_{11},$$

$$\frac{r + 38}{49} = 2 + = u_{12}, \qquad \frac{r + 60}{42} = 3 + = u_{13},$$

$$\frac{r + 66}{31} = 4 + = u_{14}, \qquad \frac{r + 58}{74} = 1 + = u_{15},$$

$$\frac{r + 16}{73} = 1 + = u_{16}, \qquad \frac{r + 57}{33} = 4 + = u_{17},$$

$$\frac{r + 75}{1} = 150 + = u_{18} = 2r.$$

As 18, the number of quotients in a complete period, is even, therefore $x = p_{18}$ and $y = q_{18}$.

$$\frac{p_1}{q_1} = \frac{75}{1}, \frac{p_2}{q_2} = \frac{301}{4}, \frac{p_3}{q_3} = \frac{376}{5}, \frac{p_4}{q_4} = \frac{677}{9}, \frac{p_5}{q_5} = \frac{3084,}{41} \frac{p_6}{q_6} = \frac{9929}{132},$$
$$\frac{p_7}{q_7} = \frac{22942}{305}, \frac{p_8}{q_8} = \frac{32871}{437}, \frac{p_9}{q_9} = \frac{121555}{1616}, \frac{p_{10}}{q_{10}} = \frac{762201}{10133}, \frac{p_{11}}{q_{11}} =$$
$$\frac{2408158}{32015}, \frac{p_{12}}{q_{12}} = \frac{3170359}{42148}, \frac{p_{13}}{q_{13}} = \frac{8748876}{116311}, \frac{p_{14}}{q_{14}} = \frac{29416987}{391081}, \frac{p_{15}}{q_{15}} =$$
$$\frac{126416824}{1680635}, \frac{p_{16}}{q_{16}} = \frac{155833811}{2071716}, \frac{p_{17}}{q_{17}} = \frac{282250635}{3752351}, \frac{p_{18}}{q_{18}} = \frac{1284836351}{17081120}.$$

$$\therefore \qquad x = 1284836351, \qquad y = 17081120.$$

I have verified these numbers.

NOTE ON THE SOLUTION OF MR. HOLBROOK'S QUESTION.

BY THE EDITOR.

Prof. Eddy (see p. 126) has given the equation of a solid whose surface is generated as described in the question proposed by Mr. Holbrook on page 72; but it appears from a subsequent clause that Mr. Holbrook had a different question in view, viz.; he asks for a "demonstration that no two ellipses can be parallel."

The surface of a solid with an elliptical base, horizontal sections of which shall be bounded by curves parallel to the periphery of the base, may be generated by a straight line which makes a constant angle with the *normal* to the ellipse while the extremity of the line describes the periphery of the ellipse. That no horizontal section, above the base of such a solid, can be an ellipse, is what we understand Mr. Holbrook to assert and desire to have demonstrated.

Horizontal sections of the solid of which Prof. Eddy has given the eqnation, are obviously ellipses; but that no section, above the base, of the solid whose surface is generated as above described, can be an ellipse, may be demonstrated as follows:

Let ABP (see diagram on next page) represent an ellipse whose semi-axes are $AC = a$ and $BC = b$, and the normal of which, at any point P, is $PO = N$. Let $A'B'P'$ be a parallel curve within the ellipse; then will the portion PP' of the normal, be of the same length for all points of the ellipse. We may therefore put $PP' = c$, a constant.